

## Designing Learning Trajectory for Teaching Fractions Using PMRI Approach with a Chessboard Context

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### ABSTRACT

The concepts of fractions and their operations is one of the materials that elementary school students need to master it. This research aims to produce student's learning trajectory in understanding the concept of fractions. The approach used is Pendidikan Matematika Realistik Indonesia (PMRI) Approach and the media used is a chessboard. The use of the chessboard context is the initial activity in learning and followed with several questions related to the context used. The research method used is design research with type of validation studies. The research subjects in this study were three students in fourth grade at Madrasah Ibtidaiyah (MI) Al Munawwarah Jambi City. This research produces a learning trajectory (LT) which contains a series of learning processes that assist students to understand the concept of fractions. Based on the results of the research, it was found that the learning trajectory using a chessboard context could assist students to understand the concepts of fractions well.

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## INTRODUCTION

Mathematics is an important subject and must be studied at all levels of education, not least at the elementary school level (Andani, Pranata, & Hamdu, 2021). Ideally, learning mathematics in elementary schools is used as the basic foundation to build student's mathematical knowledge (Lidinilah, Apriliya, Mulyasari, Andriani, & Pratiwi, 2015). (Dahlia, Pranata, & Suryana, 2020) stated that mathematics learning needs to be taught to students as their provision, including the ability to think logically, analytically, systematically, critically, creatively, and the ability to work cooperatively.

However, mathematics learning in elementary schools is still relatively low, because teachers still use conventional learning models, which make students cannot play an active role in class (Kurino, 2020). It is proven from research by Febriyanto, Haryanti, & Komalasari (2018) which states that in learning mathematics, students still play passive role in solving a problem. It is happened due to the learning is still teacher-centered, so it makes an assumption that mathematics is a difficult and boring subject. To overcome it, a learning approach is needed to make it easier and students become the main centered during learning mathematics. Learning approach used also must be oriented toward real world to make

students understand better. Learning approach oriented to the real world and centered on students is Pendidikan Matematika Realistik Indonesia (PMRI) approach.

PMRI is an adaptation of Realistic Mathematics Education (RME) learning theory from Netherlands and has been adapted with geographical conditions in Indonesia (Sembiring, 2007). In Indonesia, PMRI has been going on since 2001 and has been widely used as an effort to improve student's interests, attitudes, and learning outcomes (Zulkardi, 2009). PMRI is a learning theory that starts from the real world and all learning activities are more emphasized on student activities to seek, find, and build their own knowledge needed so that learning becomes student-centered (Putri, 2013; Sembiring, 2010; Sembiring, Hadi, & Dolk, 2008; Wijaya, Elamani, & Doorman, 2021; Zulkardi & Putri, 2010). RME learning can have a positive impact on mathematical creative thinking skills (Iskandar & Juandi, 2022). The context and learning media which is chosen must also be easy to imagine and often encountered by students (Akker, Gravemeijer, McKenney, & Nieveen, 2006; Bakker, 2004; Helsa & Hartono, 2011).

(Zulkardi & Putri, 2019) stated that there are five characteristics of PMRI. That's characteristics namely 1). Using real-world contexts as a starting point for learning mathematics, 2). Using a models as a bridge between abstract mathematics and the real world that assist students to learn mathematics at various levels of abstraction, 3). Using student's own production or strategies as a result of the mathematics they done, 4). Interaction is very important for learning mathematics between teacher and students also students and students, and 5). Connections between disciplines, both within and outside of disciplines for meaningful problems in the real world. RME or PMRI approach makes mathematics teachers easier to develop mathematical concepts and ideas starting from the real world. The real world does not mean physically and visually concrete, but also includes what the child's mind can imagine. Thus, its approach uses a real world situation or a real context as a starting point for learning mathematics (Firdaus, 2018). By using that characteristics of PMRI, in this research, it will be applied in fractions material for fourth grade students in elementary school level.

The concepts of fractions and their operations is one of the material that elementary school students need to master it (Ananda, 2018). According to Firdaus (2018), the fractions that students learn is number that represent the part of the whole which is expressed with  $a/b$ , with condition  $a$  and  $b$  are integers,  $b \neq 0$ , and  $a$  is not a multiple of  $b$ . To support the learning process, according to Andriani & Wahyudi, (2016), we must use media as a tool to support the learning process in the classroom and help foster student motivation. Ramlah, Riana, & Abadi (2022) stated that we must use media like puzzle as an instruction for student to understand materials. Also Firdaus (2018) stated that the use of learning media, especially real objects has an important role in learning mathematics in elementary schools to achieve mathematical understanding and meaning.

Several research on fractions have been carried out. Soraya, Yurniwati, Cahyana, Sumantri, & Adiansha (2018) have applied RME to elementary school students in fraction material to improve creative thinking abilities. The media used is a circular puzzle. The results obtained are that by applying RME, students become more creative in learning fractions. Furthermore, Primasari, Zulela, & Fahrurrozi (2021) have also conducted a fraction research in elementary schools using the RME approach. The media used to help students understand the material is pizza. The results obtained are that RME has an important role to produce learning trajectories in learning fractions, so that students understand the material more easily. Meanwhile, in this study, by also using the RME or PMRI approach, the media used was chessboard and colored paper and it is a difference from previous studies. The chessboard is used because it has squares (black and white) that are the same size. By

using a chessboard assisted and colored paper, students are expected to be able to understand fractions due to it's easy to imagine. This research is expected to give contributions to help teachers teach fractions material using a chessboard and colored paper, also to design a learning trajectory for teaching fractions using the PMRI approach with a chessboard context.

## METHOD

This research uses design research method with type of validation studies. The aim of this method is to develop theories of the learning process with methods designed to support learning, develop a hypothetical learning trajectory (HLT), and develop local instruction theory (LIT) through collaboration between researcher and teacher to improve the quality of learning (Cobb, Confrey, Disessa, Lehrer, & Schauble, 2003; Gravemeijer & Cobb, 2006; Plomp, 2013). The Activities in design research is a cyclical process of thought experiment and instruction experiment (Gravemeijer, 1994). There are three reasons why using design research, including 1). increasing the relevance of research, 2). developing the theoretical basis empirically, and 3) determining the robustness of the application of the design. Design research also has five characteristics, namely: 1). interventionist nature, 2). proses oriented; 3). reflective component; 4). cyclic character; 5). theory oriented (Plomp, 2013).

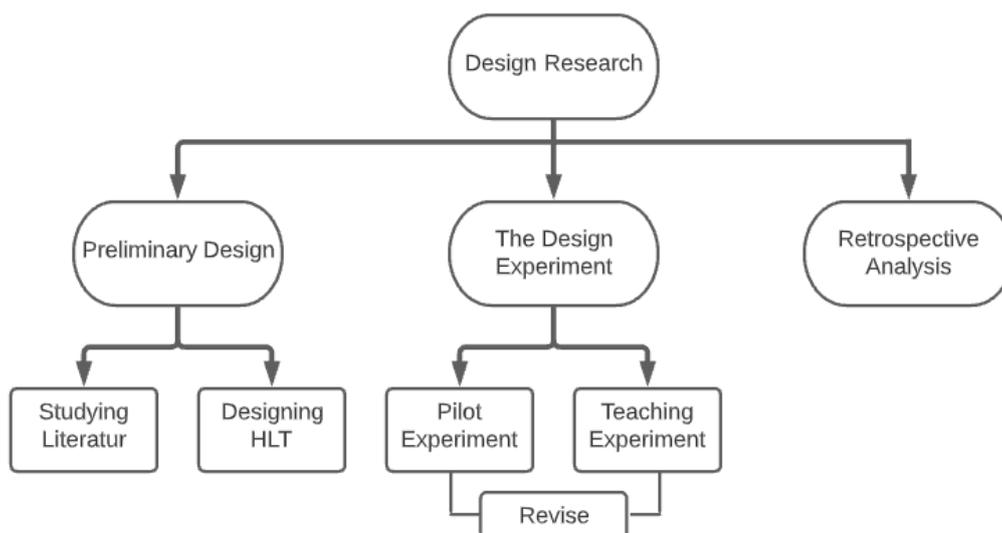


Figure 1. The flow of design research with type of validation studies

There are three stages in design research, including preparing for the experiment (preliminary design); the design experiment (pilot experiment and teaching experiment); and the retrospective analysis (Reeves, Herrington, & Olivier, 2004; van den Heuvel-Panhuizen, 2003). However, in this research, it was only limited to the pilot experiment stage. This research was conducted at Madrasah Ibtidaiyah (MI) Al Munawwarah Jambi City on October 2021. The research subjects are three students of fourth grade selected based on their level of mathematical ability. In this study, the data collection techniques used were observation, documentation, and student worksheets.

## RESULTS AND DISCUSSION

### A. *Preliminary Design*

In preliminary design stage, researcher studies about the curriculum and literature. The curriculum in Indonesia currently uses the Curriculum 2013 (K13), which previously used the Tingkat Satuan Pendidikan Curriculum (KTSP). The change from KTSP to K13 has an impact on learning activities in the classroom which were previously teacher-centered to become student-centered (Bamega, Sulasmono, & Setyaningtyas, 2019). According to Mulyasa (2013) in K13, students are actively involved in learning activities due to they are the center of learning activities that lead to make competences and characters. K13 uses a scientific approach in learning, it's mean that students are asked to observe, making a questions, collecting information, analyzing information, and communicating the learning outcomes. So that, the role of students is more active in learning than teacher. Teacher only assists students in finding material and developing skills that can be used to solve everyday life problems.

This is same way with PMRI principles. Zulkardi & Putri (2010) stated that PMRI has three principles in accordance with RME, namely:

#### 1) Guided rediscovery and didactic phenomenology

Since mathematics in learning RME is a human activity, so guided reinvention can be interpreted that students should be given the opportunity to experience the same process when mathematics was discovered. This principle can be inspired by using informal procedures. This effort will be achieved if the teaching is carried out using situations in the form of phenomena that contain mathematical concepts and student's real activities.

#### 2) Progressive mathematization

Situations that contain phenomena that are used as materials and applications in teaching mathematics must be graded from the real situation before reaching the level of formal mathematics. In this case, two kinds of mathematization must be used as the basis for departing from the real level of learning mathematics to the level of formal mathematics.

#### 3) Self-developed models

The role of self-developed models is a bridge for students from real situations to concrete situations or from informal mathematics to formal mathematics. That is, students make their own models in solving problems. The first is a model of situation that's close to the daily life of students. Through generalization and formalization, the model is converted into a "model-of" of the problem. The "model-of" will shift to a "model-for" in similar problem (formal mathematical problem). It will eventually become a model in formal mathematics.



**Figure 2. Discussion with mathematic teacher**

After studying the curriculum, researcher then made observations to school and discussed directly with teacher about literature to make HLT and learning activities. In making the HLT with teacher, researcher designed questions using the PMRI approach which is oriented to the real world. The media used is chessboard and colored paper. There are two types of questions designed. First, arrange the fractions using prepared media. It aims to help students to understand the value of fractions. Then the second question is given a fractional value, then students are free to express their answers. It aims to make students think creatively using various operations on fractions. The HLT that has been made can be seen in table 1. And for teaching fraction, researcher uses video to make student understand how to use chessboard and colored paper as Figure 3.

**Table 1. HLT for teaching fraction using chessboard context**

<b>Activities</b>	<b>Main Purpose</b>	<b>Activity Description</b>
Giving fraction questions	Students understand fraction problems.	The teacher gives a fractions worksheet along with a chessboard and colored paper.
Student use of chessboard and colored paper	Students are able to use a chessboard and colored paper as a media to solve fraction problems.	The teacher guides students through video to use the chessboard and colored paper as a media for learning fractions.
Solving the problem of fraction	Students are able to solve all fraction problems.	The teacher encourages students to solve fraction problems and discuss it together.

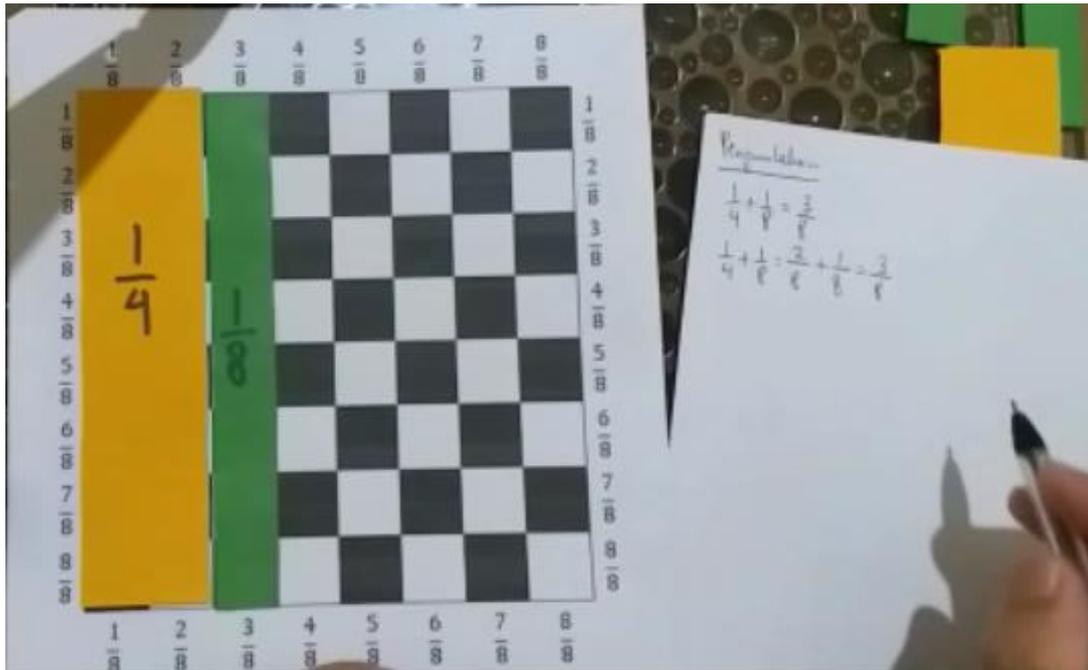
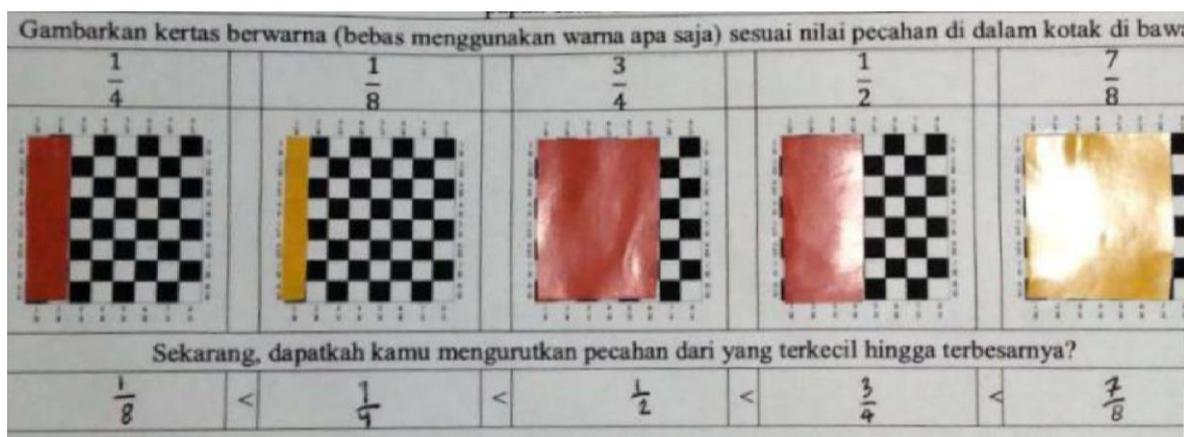


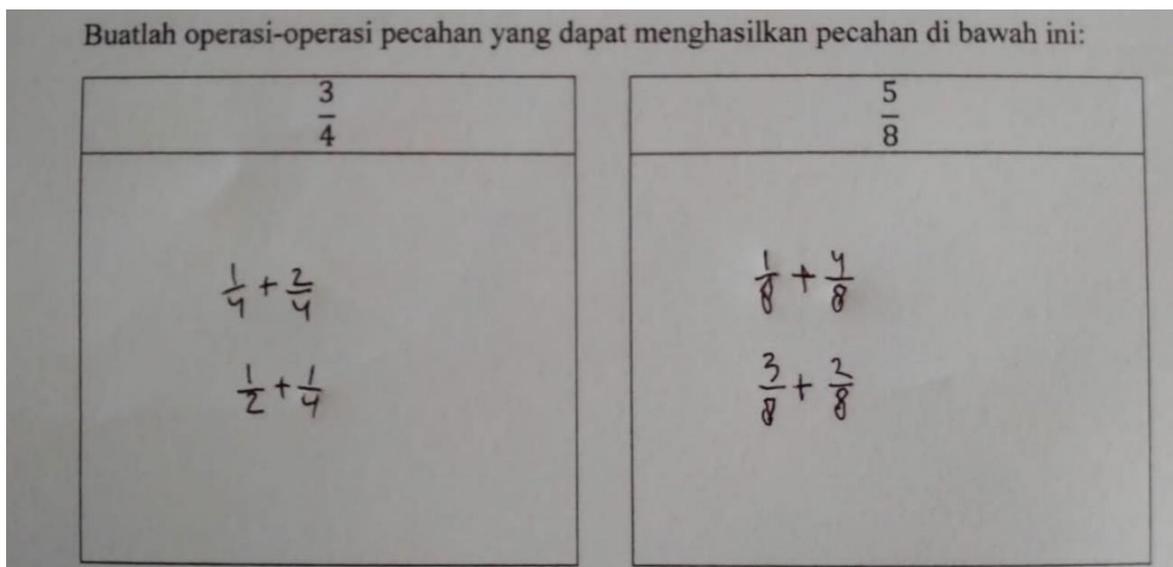
Figure 3. Teaching fraction using video

After giving video to students, then researcher conducts a pre-test with three students to review the effectiveness of media, worksheet, and HLT that had been made.

**B. Pilot Experiment**

In pilot experiment stage, three students in fourth grade selected to complete the fraction problems using prepared media. By using HLT before, presentation of material starts from explaining how to use chessboard and colored paper (with video) to lead students to answer the questions that have been given. There are two questions, in part A, students are asked to arrange some fraction from smallest to largest. In part B, given a fraction then students are asked to answer using some operations. The following is one of student’s answer during pilot experiment.





**Figure 4. Student's answers during the pilot experiment**

From the student's answers, in part A, student can arrange fraction using cheeseboard and colored paper from lowest to largest correctly. Also in part B, student can make some operation of fraction correctly to answer the question but still use addition. It's probably because the fraction additional operation is easier than the other operations. The other students also answer in part B with fraction additional operation. But overall of three students, they can answer questions correctly. In this research, research has been limited in this stage.

## CONCLUSIONS

There are several stages in this research. In preliminary design stage, researcher studies about the curriculum and literature. After that, researcher and teacher collaboratively make a HLT as a guide in learning fraction. In this research, we use chessboard and colored paper as media for learning fraction. The use of the chessboard context is the initial activity in learning and followed with several questions related to the context used. The use of this context aims to lead students to understand the concept of fractions.

In pilot experiment stage, three students in fourth grade selected to complete the fraction problems. There are two questions, in part A, students are asked to arrange some fraction from smallest to largest, and in part B, given a fraction then students are asked to answer using some operations. All of students can answer questions correctly.

This research also produces a learning trajectory (LT) which contains a series of learning processes to assist students understand the concept of fractions. Based on the results and discussions that have been described, it can be concluded that the learning trajectory using a chessboard context can assist students to understand the concept of fractions well.

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## Mendesain Lintasan Belajar untuk Pengajaran Pecahan Menggunakan Pendekatan PMRI dengan Konteks Papan Catur

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### Abstrak

Konsep pecahan dan operasinya merupakan salah satu materi yang perlu dikuasai oleh siswa sekolah dasar (SD). Penelitian ini bertujuan untuk menghasilkan lintasan belajar siswa dalam memahami konsep pecahan. Pendekatan yang digunakan adalah Pendekatan Pendidikan Matematika Realistik Indonesia (PMRI) dan media yang digunakan adalah papan catur. Penggunaan konteks papan catur merupakan kegiatan awal dalam pembelajaran serta diikuti beberapa pertanyaan terkait dengan konteks yang digunakan. Metode penelitian yang digunakan yaitu *design research* tipe *validation studies*. Subjek penelitian dalam penelitian ini adalah tiga siswa kelas IV Madrasah Ibtidaiyah (MI) Al Munawwarah Kota Jambi. Penelitian ini menghasilkan *learning trajectory* (LT) yang memuat rangkaian proses pembelajaran yang membantu siswa memahami

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konsep pecahan. Berdasarkan hasil penelitian, ditemukan bahwa lintasan belajar menggunakan konteks papan catur dapat membantu siswa memahami konsep-konsep pecahan dengan baik.

**Keywords:** *Learning Trajectory*, Pecahan, PMRI, Papan Catur

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