Representation Of Students' Relational Understanding In Solving Evidence-Related Problems

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ABSTRACT

The goal of this study is to see how students' relational understanding affects their ability to solve proof problems in basic logic classes. Because this study will illustrate how students' relational knowledge in solving arithmetic problems, it is a qualitative research with a descriptive kind. Tests and interviews were used to gather data, which was then evaluated using research data reduction methodologies, which included presenting data in the form of descriptions based on features of relational knowledge, and finally concluding the findings. The findings revealed that students' relational understanding appeared to be carried out in solving proof problems, specifically in the interpretation aspect, then in the aspect of preparing questions and hypotheses, and finally in the aspect of generalization, all of which were visible when observations were made and were found in the responses of students. However, there are still flaws and inconsistencies in the signs of applying and performing reasoning.

INTRODUCTION

The importance of learning objectives cannot be overstated (Pane & Dasopang, 2017). This aim will be met if students comprehend what the teacher has learned and communicated. As a result, comprehension is critical in learning (Kusmaryono, 2014). Involved in the study of mathematics in high school and college. The knowledge of concepts, for example, is an example of this type of understanding (Susanti & Taufik, 2021). Students can solve arithmetic problems more easily if they understand the appropriate principles. (Booth, 2014). There is also a relational understanding that is highly crucial in studying mathematics in addition to knowing concepts.

Understanding what to do and why is referred to as relational understanding (Skemp, 2020). Relational understanding, on the other hand, can be defined as pupils' ability to figure out how procedures will be employed to solve problems and why each step in the technique is carried out. As a result, having a relational awareness is critical for pupils. The procedural category for indicators of skill and fluency in carrying out processes and the proper results is the relational understanding that is the subject of this research. Recognizing when to use procedures, knowing the prerequisites for employing procedures, and being able to present logical arguments are the conceptual categories for indicators.

Relational understanding will be able to improve students' understanding of learning mathematics by focusing on how the process will be employed (Syarifah, 2017). In order to see relationship comprehension, a unique teaching style is required (Patkin & Plaksin, 2019). A notion must be constructed from a principle that produces many plans in relational understanding (Star, 2020). It takes a long time to figure out why this is. From relational understanding, various things are produced, including right knowledge, training to see the problem as a whole, developing skills, and building inductive capacities (Maulyda, 2020). According to the reasoning above, it is critical to teach students relational understanding, which includes learning about proof material in the Basic Logic course. The proof material in the Logic course requires a thorough comprehension, as well as clear methods and easy-to-understand procedures, to ensure that there are no errors in the proof. Interpretation, asking questions and hypotheses, and generating generalizations are all facets of relational understanding that were used in this study.

Students' knowledge of the proof rules content was found to be highly inadequate, and students did not comprehend why, based on observations of students in the Basic Logic Course. Students, for example, do not understand when to use the norms of exchange, deriving conclusions, and conditional proof. Lack of understanding of what will be demonstrated, the method of evidence that will be used, lack of capacity to manipulate what is known and what method will be utilized, and lack of awareness of the procedures to be used are all factors that contribute to students' difficulties in proving (Salsabila et al., 2016). a study conducted by (Siregar, 2016) Students have trouble demonstrating not just because of a lack of competence, but also because of a lack of practice working on proof questions, according to the author.

The study of relational understanding was conducted by (Sulasmi et al., 2020) which claims that educators play a critical role in learning, particularly when it comes to relational knowledge. (Yazidah et al., 2018) In his research, he discovered that the process of relational understanding is good, but that it may be improved by utilizing suitable teaching approaches. (Wulandari & Rakhmawati, 2019) additionally claims that children who are educated utilizing particular methodologies have different relational knowledge.

The researcher will analyze the relational understanding in proof as a result of the foregoing exposition of the problem's history. It is intended that by providing a description of the relational understanding of the rules of proof, it may aid in the teaching of the rules of proof to students. The challenge in this study is how the students' relational understanding is used to solve the problem of proof rules. While the goal of the study is to describe students' relational grasp of the rules of proof when addressing problems

METHOD

The research method employed in this study was a qualitative approach with descriptive research. This study will look at how students use relational understanding to solve problems involving proof rules. The information gathered throughout the research will be examined and summarized using relational understanding indicators.

The participants in this study were mathematics education students who studied Basic Logic courses. The goal of the study is to see how students' relational understanding affects their ability to solve proof problems. The research procedure begins with the preparation stage, which includes the analysis of relational understanding indicators and the preparation of instrument questions; the second stage is the implementation stage, which includes giving students test questions and conducting interviews; and the final stage is data analysis, which involves the researcher receiving the data and analyzing it in accordance with the relational understanding indicators that have been prepared.

Tests and interviews are two methods for gathering information. The purpose of the test is to determine how the students' relational understanding fared. While interviews are used to determine what the reasons are or how students obtain or use the information supplied, they are also used to determine how students obtain or use the information presented. A test sheet was employed in this investigation as an instrument. The purpose of this study's data analysis is to look at the results of student work, which are then compared to indications of relational comprehension. The results are then explained, and the findings from the interview activities are used to support and strengthen them. The following are the features and indicators used in the analysis of test findings.

Table 1. Indicators of Relational Understanding

Aspect	Indicator
Interpretation	Performing an Identification
	Apply
Posing questions and formulating hypotheses	Posing questions
	Formulating hypotheses
Generalization	Reasoning
	Thinking in a Flexible Way
	Conclude

RESULTS AND DISCUSSION

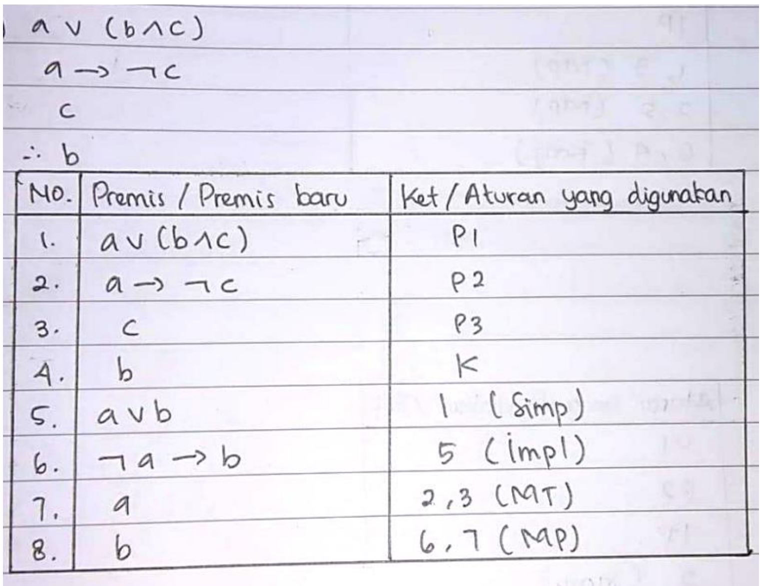
This study was done at the University of Muhammadiyah Malang's Mathematics Education Study Program, where students studied Basic Logic classes on the subject of proof rules. The findings of the study or test results are assessed using features and signs of Relational Understanding, such as interpretation, asking questions and hypotheses, and making broad generalizations. The next talk will go over the consequences of relational understanding in depth.

a) Interpretation

The interpretation factor is intended to assess students' ability to interpret and determine what is known in the problem. Students will be able to proceed to complete the proof questions once they have figured out how to interpret or identify the questions. There are signs to detect and apply in the interpretation section.

Performing an Identification

Indicators for identification can be noticed in the way pupils express what they know, such as laying out the known premises and what conclusions will be sought while completing this proof rule. Students that score high on this indicator can write and do it well. The following is an example of student work on proof rules (see Figure 1).


$a \vee (b \wedge c)$
$a \rightarrow \neg c$
c
$\therefore b$

No.	Premis / Premis baru	Ket / Aturan yang digunakan
1.	$a \vee (b \wedge c)$	P1
2.	$a \rightarrow \neg c$	P2
3.	c	P3
4.	b	K
5.	$a \vee b$	1 (Simp)
6.	$\neg a \rightarrow b$	5 (Impl)
7.	a	2, 3 (MT)
8.	b	6, 7 (MP)

Figure 1. Student Activity 1

Figure 1 illustrates students' ability to write out the problem's premises as well as the conclusions that will be addressed in the proof problem (finishing steps 1, 2, 3, and 4). is to show that the three premises will lead to the conclusion "b." The next stage is to prove these premises after they have been written down.

Apply

One part of interpretation is the use of indicators. When students are able to apply the rules from the proof to the solution of the proof problem, this is an example of this signal. Students are claimed to be able to perform it in this indicator, even though some students have difficulty and do not complete it correctly. Figure 2 shows an example of student completion in the problem of the rules of proof.

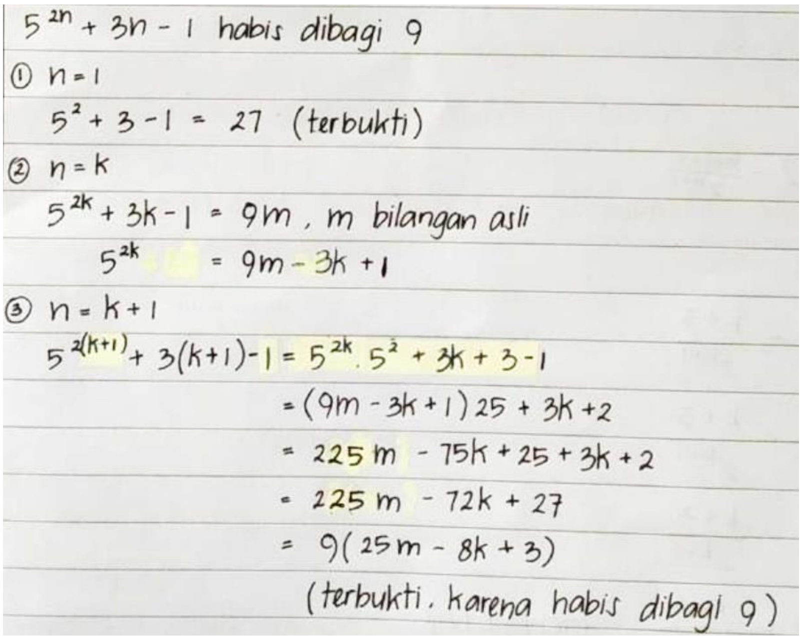

$5^{2n} + 3n - 1$ habis dibagi 9
① $n = 1$
$5^2 + 3 - 1 = 27$ (terbukti)
② $n = k$
$5^{2k} + 3k - 1 = 9m$, m bilangan asli
$5^{2k} = 9m - 3k + 1$
③ $n = k + 1$
$5^{2(k+1)} + 3(k+1) - 1 = 5^{2k} \cdot 5^2 + 3k + 3 - 1$
$= (9m - 3k + 1) 25 + 3k + 2$
$= 225m - 75k + 25 + 3k + 2$
$= 225m - 72k + 27$
$= 9(25m - 8k + 3)$
(terbukti, karena habis dibagi 9)

Figure 2. Student activity 2

Students use the induction rule to answer proof issues in mathematics, as shown in Figure 2. In the induction rule, students must meet three conditions during the proof process: proving $n = 1$, proving $n = k$, and proving $n = k + 1$. Students are able to solve this problem by applying mathematical induction methods and completing and proving the three conditions supplied. Not only that, but it has also employed multiple solution steps from algebra's solving rules to prove it. For instance, in the last step, use the distributive property of multiplication to arrive at the conclusion that will be addressed in the problem.

b) Posing questions and formulating hypotheses

Indicators for asking questions and providing hypotheses on questions are included in the aspects of asking questions and hypotheses. This is demonstrated by conducting interviews with students who are currently working on the proof problem. The two indications are explained in the following paragraphs.

Posing questions

Students can observe and assess the existing questions in the indicator for asking this question. Because it is seen when students respond to the questions posed during the interview. The following dialogue appears to be taking place:

T : *What exactly does the question imply?*
M : *The challenge wants you to show that if you know a, then an or b, then c, and that the conclusion is a and c bu if you know a.*

From the dialogue, it appears that the kids have grasped the questions and are capable of asking follow-up questions. In the proving process, at the very least, students must know ahead of time which path the evidence will go from the problem, making it easier to solve.

formulating hypotheses

Proposing a hypothesis is the next step in this process. Students can do it before starting the proofing procedure in this indicator. This can also be evident in the interactions amongst students who are working on them during interviews.

T : *How does M's photo go through the proofing process?*
M : *I believe that in order to prove ma'am, I should employ a well-known assumption.*
T : *Isn't that a command to rely on circumstantial evidence?*
M : *Yes, ma'am, I'll have to apply the indirect evidence procedure outlined in the guidelines.*
T : *How do you prove it in a roundabout way? Isn't it different?*
M : *It's different, ma'am, because we may utilize the conclusion as a new premise by employing its negation, and then if it's indirect evidence, we'll see if there's a conjunction of one of the identical statements to the negative or not, allowing us to determine the direction of proof from indirect evidence.*

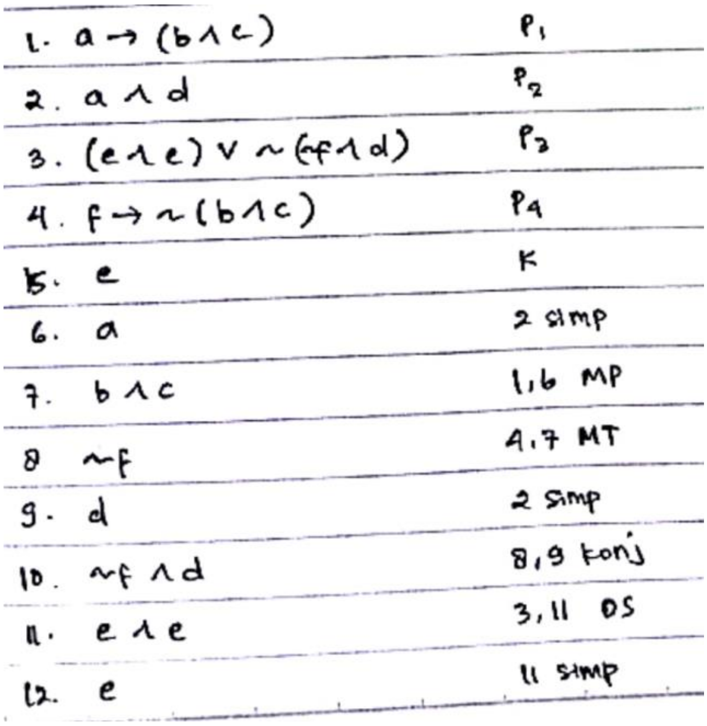
Students were able to explain the needs for indirect evidence during the discussion. The criterion is that the conclusion's negation is utilized as a new premise, which is shown by the presence of a conjunction of the same statement and its negation in indirect proof.

c) Generalization

The generalization part of proving is an aspect of reasoning that allows students to discover what is created by the proof process. Reasoning indicators, flexible thinking indicators, and concluding indicators are all part of the generalization aspect. In the following conversation, more information on the topic of generalizing will be provided.

Reasoning

The reasoning indication is the first indicator in the generalizing category. The most essential signal is the reasoning indicator. It is said to be crucial because it is what completes the verification process or determines whether what is in the inquiry is proven or not in this reasoning indicator. Of course, you utilize or must comprehend a variety of additional indications in the verification process, such as implementing indicators. Because you must employ some mathematical rules. Students are able to accomplish it and complete it in this indicator, despite the fact that some students do it with flaws and mistakes. An example of student work is shown below.



1. $a \to (b \wedge c)$	P1
2. $a \wedge d$	P2
3. $(e \wedge e) \vee \sim(f \wedge d)$	P3
4. $f \to \sim(b \wedge c)$	P4
5. e	K
6. a	2 simp
7. $b \wedge c$	1,6 MP
8 $\sim f$	4,7 MT
9. d	2 simp
10. $\sim f \wedge d$	8,9 konj
11. $e \wedge e$	3,11 DS
12. e	11 simp

Figure 3. Student activity 3

Figure 3 illustrates this point. This is an example of applying the rules of exchange and inference to complete the proof. It will be demonstrated in the problem that the conclusion is e, based on premises 1, 2, 3, and 4. Of course, the student has employed the rules in the rules of evidence, notably various exchange rules and inference rules, to reach his conclusion. In addition, students have employed all of the primary premises in the proof procedure, namely premises 1, 2, 3, and 4. Not only that, but students also employ all of the other premises that have been formed by applying some of the accessible rules in order to arrive at the problem's conclusions and be proven. It may be argued that students are able to reason well in addressing problems involving proof rules since they are able to apply the rules that exist in the proof and employ existing premises. Even though it was discovered that some students did not do it since they only utilized a few premises to establish it, it was still discovered that some students did not do it.

Thinking in a Flexible Way

In order to solve mathematical issues, you must be able to think in a flexible manner. In this scenario, open thinking is meant by flexible thinking. Because it will be possible to choose or utilize many methods to tackle these mathematical difficulties if you think honestly. When it comes to proofing, flexibility is also essential. Because it will be clear how to arrive at the desired conclusions with flexible thinking. Students have also used this indicator, albeit most of them have had problems using it. Students' challenges are due to a lack of students conducting exercises from a variety of questions, as a result of which they seldom use all of the available rules and only employ a few of them; here is an example of flexible thinking in the rules of evidence.

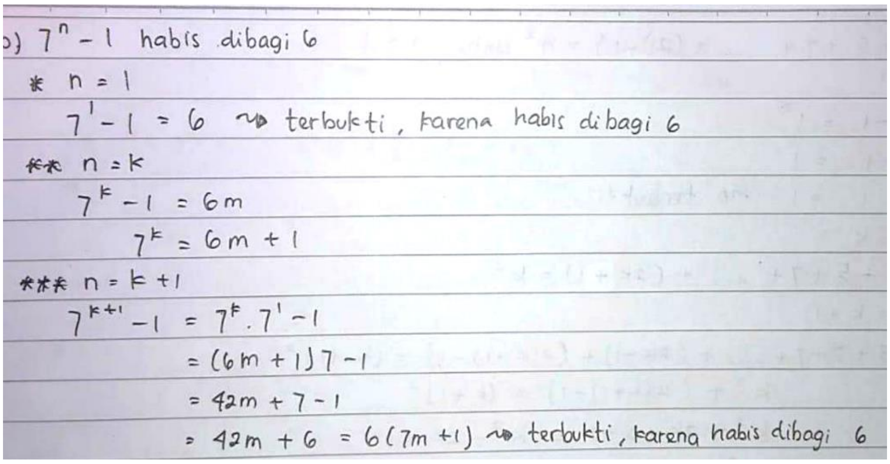


Figure 4. Student Activity 4

In order to solve the proof problem, students used flexible thinking in Figure 4. Students utilize the power rule and then the rules in algebraic operations to prove that the question in question can be divided by six in the proof of $n = k + 1$. Even though there are alternative approaches that can be used, such as logarithmic rules or something else.

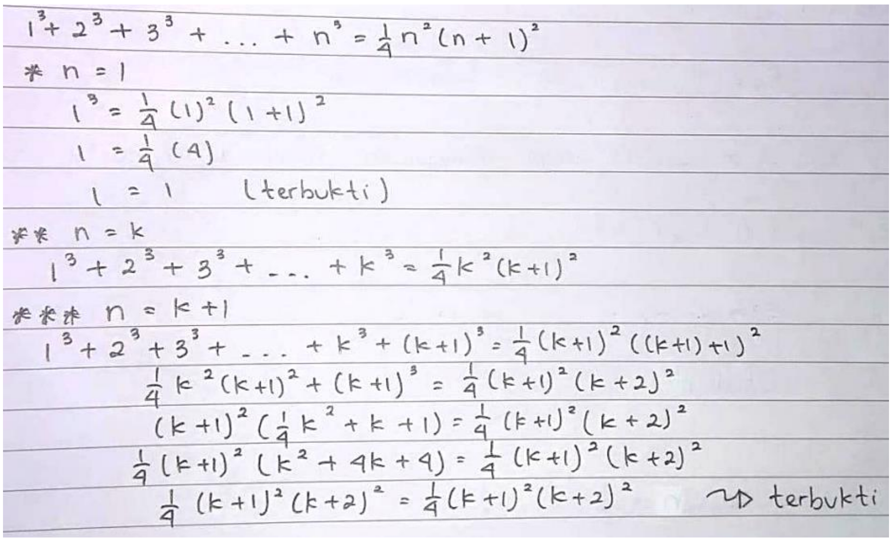


Figure 5. Student Activity 5

Figure 5 shows the second example of flexible thinking. Students should be able to produce proofs aimed at the right-hand side and proofs aimed at the right-hand side in the figure for the third step, namely $n = k + 1$. As a result, students can select from a list of equations that are considered simple to answer. Or, to put it another way, students can change the direction of the proof as they see fit..

Conclude

The final phase in the generalization process is to come to a conclusion. In this situation, the learner is able to write the ultimate conclusion of the issue posed, namely whether or not the question is proven after numerous methods or phases of evidence have been completed. Students have completed this indication and recorded the conclusions of each question so that it can be determined whether the method was proven or not.

Based on the study's results, it can be concluded that students' relational understanding in addressing proof problems is visible and has been carried out by students during the proof process. It's just that some students continue to make mistakes or are less accurate in their solutions, particularly on the indicators of application and reasoning. This is due to the fact that applying indications and reasoning necessitates precision and skills in solving them or applying other mathematical principles, such as powers, multiplication, or number operations in other algebra.

The study of relational understanding was carried out by (Rahmad et al., 2016) also those who arrive at the conclusion that visual and symbolic representations can be employed to investigate relational knowledge. So it can be argued that in order to teach proof in mathematics later, visual representations and dimbolik can be used or paid attention to so that the proving process runs easily and without complications. Although, when seen from a relational perspective, the process of proof may be seen in this study.

This study contradicts the findings of another study (Mardiana & Hidayanto, 2016), because it is asserted that pupils' comprehension is still instrumental or non-relational because students are unable to describe the rationale This can be due to variances in material, but it can also be due to the fact that when students grasp the rules that exist in the rules of evidence, they will be able to accomplish it simply. Furthermore, using difficult operations in calculations, such as those in the derivative idea, is not recommended by the principles of proof.

CONCLUSION

According to the findings, kids already have a relational understanding and have demonstrated their ability to solve proofs in fundamental logic tasks. Both mathematical induction and conditional proof rules are used in the proof. Although some people still make mistakes and aren't exact because of inaccuracies, this relational knowledge is widely accepted. Students have legitimate motives for using the rules of forming conclusions and rules of trading that are already in place. As a result, it looks that the relationship understanding is complete.

REFERENCES

Booth, J. L. (2014). Why Can ' t Students Get the Concept of Math ? In *the International Dyslexia Association* (Issue June).

Kusmaryono. (2014). The Importance Of Mathematical Power In Mathematics Learning. *International Conference on Mathematics, Science, and Education*, *2014*(Icmse).

Mardiana, S., & Hidayanto, E. (2016). Pemahaman Instrumental Dan Relasional. *Prosiding Seminar Nasional Matematika Dan Pendidikan Matematika FKIP UNS*, *November*, 67–76.

Maulyda, M. A. (2020). *Paradigma Pembelajaran Matematika Berbasis NCTM* (Issue 16 Januari 2020). CV. IRDH Malang.

Pane, A., & Dasopang, M. D. (2017). Belajar Dan Pembelajaran. *FITRAH:Jurnal Kajian*

Ilmu-Ilmu Keislaman, *3*(2), 333. https://doi.org/10.24952/fitrah.v3i2.945

Patkin, D., & Plaksin, O. (2019). Procedural and relational understanding of pre-service mathematics teachers regarding spatial perception of angles in pyramids. *International Journal of Mathematical Education in Science and Technology*, *50*(1), 121–140. https://doi.org/10.1080/0020739X.2018.1480808

Rahmad, B. A., Ipung, Y., Abdur, R. A., Sisworo, & Dwi, R. (2016). Mathematical representation by students in building relational understanding on concepts of area and perimeter of rectangle. *Educational Research and Reviews*, *11*(21), 2002–2008. https://doi.org/10.5897/err2016.2813

Salsabila, E., Ratnaningsih, R., & Hadi, I. (2016). Pembekalan Pemahaman Metode Pembuktian Matematika dan Penerapan Strategi Abduktif-Deduktif Untuk Mengembangkan Kemampuan Membuktikan Konsep Aljabar Abstrak Pada Mahasiswa Jurusan Matematika FMIPA UNJ. *Jurnal Matematika Integratif*, *11*(1), 15. https://doi.org/10.24198/jmi.v11i1.9392

Siregar, I. (2016). Masalah Pembelajaran Pembuktian Matematika Bagi Mahasiswa Di Indonesia. *Jurnal Pendidikan Matematika STKIP Garut*, *5*(3), 315–324.

Skemp, R. R. (2020). Relational Understanding and Instrumental Understanding. *The Arithmetic Teacher*, *26*(3), 9–15. https://doi.org/10.5951/at.26.3.0009

Star, J. (2020). Instrumental and Relational Understanding in Mathematics Education. In *Encyclopedia of Mathematics Education*. https://doi.org/10.1007/978-3-030-15789-0_79

Sulasmi, S., Sampoerno, P. D., & Noornia, A. (2020). Development of learning using Indonesian realistic mathematics education approach to build students' relational understanding of derivative. *Journal of Physics: Conference Series*, *1470*(1). https://doi.org/10.1088/1742-6596/1470/1/012060

Susanti, R. D., & Taufik, M. (2021). Analysis of Student Computational Thinking in Solving Social Statistics Problems. *SJME (Supremum Journal of Mathematics Education)*, *5*(1), 22–31. https://doi.org/10.35706/sjme.v5i1.4376

Syarifah, L. L. (2017). Analisis Kemampuan Pemahaman Matematis Pada Mata Kuliah Pembelajaran Matematika Sma Ii. *Jurnal Penelitian Dan Pembelajaran Matematika*, *10*(2), 57–71. https://doi.org/10.30870/jppm.v10i2.2031

Wulandari, F., & Rakhmawati, R. (2019). Analisis Kemampuan Pemahaman Relasional Matematis : Dampak Strategi Pembelajaran Index Card Match. *Desimal: Jurnal Matematika*, *2*(3). https://doi.org/10.24042/djm.v2i3.4291

Yazidah, N. I., Argarini, D. F., & Sulistyorini, Y. (2018). Analysis on Relational-Understanding in Solving Problem for Field Independent Students. *Al-Ta Lim Journal*, *25*(3), 272–280. https://doi.org/10.15548/jt.v25i3.502